Metacognitive Activity and Collaborative Interactions in the Mathematics Classroom: A Case Study

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Abstract.
Previous research on the development of metacognitive skills has used Vygotsky's notion of the zone of proximal development, concentrating particularly on teacher-student interaction. However, Vygotsky also conceptualised the ZPD as allowing for peer interaction, so that students might monitor and extend each other's thinking. This exploratory study, carried out in a Year 11 mathematics classroom, provides evidence of metacognitive strategy use during informal peer interaction, and identifies a collaborative discussion style that was the vehicle for metacognitive activity.

Recently published Australian curriculum documents have drawn attention to two important aspects of mathematics learning: *collaborating with peers* in order to construct, justify and critique mathematical ideas, and *developing metacognitive procedures* for making plans, checking progress and evaluating outcomes (e.g. Australian Education Council, 1991; Board of Senior Secondary School Studies, 1992). However, the processes involved in collaborative learning and metacognitive self-regulation are not well understood (Good, Mulryan and McCaslin, 1992; Schoenfeld, 1992); moreover, peer interaction may even interfere with metacognitive decision making (Goos, 1994). This paper reports on the early stages of a study that will attempt to identify the features of collaborative interactions that mediate metacognitive development in secondary school mathematics students.

Theoretical Background

In recent years researchers interested in improving students' metacognitive capabilities have drawn on Vygotsky's (1978) sociocultural theories of learning, which claim that higher mental processes have their origins in social interactions with either expert adults or peers. Students' individual self-regulatory capacities may be extended if they initially operate within a zone of proximal development, where interaction with others elicits emerging intellectual skills while offering support in the form of other-regulation, or 'vicarious consciousness' (Bruner, 1985). The teacher's role in providing *adult guidance* has been the subject of considerable theorising (e.g. Bruner, 1985; Wertsch, 1984), and has been exemplified in Schoenfeld's (1985) work with college level mathematics students. However, less is known about the processes of *peer collaboration* that might contribute to metacognitive development.

Although there is a large body of literature devoted to peer learning (see Good, Mulryan and McCaslin, 1992, for a review), it is important to note that not all forms of peer interaction can be classed as collaborative (Damon and Phelps, 1989). The hallmark of collaboration is mutuality—exploring each other's reasoning and viewpoints in order to construct a shared understanding of the task (Granott, 1993). Producing mutually acceptable solution methods and interpretations entails reciprocal interaction, requiring students to propose and defend their own ideas, and to ask their peers to clarify and justify any ideas they do not understand (Cobb, Wood and Yackel, 1991).

Despite the potential for collaborative settings to provide a natural forum in which students could monitor their own and each other's thinking, small group work in mathematics does not always produce metacognitive benefits (Stacey, 1992). The purpose of the pilot study reported here was to observe students' thinking and social interactions in order to identify examples of metacognition and collaboration in action—that is, in a normal classroom environment. Three research questions were addressed:

1. What evidence is there that students use metacognitive strategies when working on classroom mathematics tasks?
2. How are metacognitive processes embedded in the social interactions that occur between students, and between students and teacher?
3. To what extent are these interactions collaborative?

Classroom Context

The study group consisted of 15 students, 7 female and 8 male. The students were participating in a Mathematics C course which formed part of their secondary accreditation. To enrol students were requested to satisfy the criterion of reaching a 'High Level of Achievement' in junior school mathematics studies. As a group, therefore, these students could be considered to be mathematically able and highly motivated.

Instruction at a global level was teacher directed. The teacher defined the task requirements for any one lesson but students were then largely responsible for the direction they chose to achieve these predetermined goals. Students were strongly encouraged to interact with the teacher and each other when working on set tasks and appeared to respond equally well to peer tutoring as direct interaction with the teacher.

The topic being examined by students was part of an introductory unit of work based on Chaos theory. This was a school option chosen and developed by the class teacher. The unit was chosen as a vehicle to:

- introduce students to an area of recently developed mathematics demonstrating the developmental nature of the discipline
- provide students with a learning experience based on the study of a topic within discrete mathematics, a branch of applicable mathematics of developing importance
- act as a vehicle for the natural use of computer technology as a means of exploring mathematical ideas.

Classes were conducted in a school computer room housing 15 Apple Macintosh LCII machines, each with access to the spreadsheet module of the software package Clarisworks. Students had gained familiarity with using these machines in junior mathematics studies but were relatively inexperienced with the use of spreadsheets and the process of iteration. The number of machines allowed each student individual access to a computer. Taken together, these factors appeared to stimulate discussion on how to complete set tasks.

The principal subjects of this study, Belinda and Louise, were chosen because of their ability and willingness to articulate their findings and difficulties as they explored ideas outlined in the unit of work. Belinda was of particular interest because of a demonstrated determination to understand ideas at a deep level. Never being satisfied with mere mastery of an algorithm, she always expressed a desire to know 'why', not just 'how'.

Method

Procedure

A small tape recorder was placed beside the two target students, Belinda and Louise, as they worked on the problem set out below while an observer (the first author) took field notes. A partial record of conversations with the teacher (the second author) and a third student, Rob, was also obtained.

Nga is offered the following terms with two different financial institutions:

Institution 1: 12.5% compounded annually
Institution 2: 12% compounded monthly

Nga knew that in the short term the conditions offered by institution 1 were superior but suspected that in the long term institution 2 might be the best proposition. For what period of time would she need to invest with institution 2 before she realised a better return on her investment.

Students were expected to construct a four column spreadsheet for each institution (Figure 1) in order to compare their respective returns. Note that the spreadsheet for Institution 2 differs from that for Institution 1 in two ways: each compounding step represents months, rather than years (Column A); and the effective interest rate per compounding period needs to be calculated by dividing the 'per annum' rate by twelve (Column C).

Institution 1

	A	B	C	D
1	Year	Principal	Interest	Amount
2	1	1000	=B2*0.125	=B2+C2
3	=A2+1	=D2	fill down	fill downm
4	fill down to *n* yrs	fill down		

Institution 2

	A	B	C	D
1	Month	Principal	Interest	Amount
2	1	1000	=B2*0.01	=B2+C2
3	=A2+1	=D2	fill down	fill down
4	fill down to 12*n* mths	fill down		

Figure 1. Spreadsheet solutions for the interest rate problem

Data Coding and Analysis

The audiotape was transcribed and the resulting protocol divided into segments, each of which represented a distinct stage in Belinda's progress towards a solution. Analyses were conducted at two levels. The first, macroscopic, level focussed on Belinda's metacognitive strategies such as planning, monitoring and evaluating, and dealing with obstacles. At the second level of analysis, conversational turns of all speakers were coded to identify their metacognitive function and contribution to the collaborative structure of the interaction. A coding scheme developed in an earlier study (Goos, 1994) was used to identify two types of metacognitive decision points: exploiting one's knowledge by proposing a *New Idea*, and keeping track of progress by making an *Assessment* of a strategy, a result, or one's knowledge or understanding. A measure of collaboration was derived by coding the transactive quality of the dialogue. Transactive dialogue is defined as discussion in which an individual's reasoning operates on a partner's reasoning, or significantly clarifies his or her own reasoning, by offering or eliciting critiques, clarifications, elaborations or justifications (Kruger, 1993). Three types of transacts were coded: spontaneously produced transactive *statements* and *questions*, and passive *responses* to transactive questions. The orientation of each transact was also noted: operations on one's partner's ideas were labelled *other-oriented*, while reasoning directed at one's own ideas was coded as *self-oriented*. This procedure produced six transaction codes: (three types) × (two orientations). The codes were then grouped to describe a collaborative discussion style that incorporates three elements of mutuality:

Self-disclosure

Self-oriented statements and responses that clarify, extend, evaluate, or justify one's own thinking.

Feedback request

Self-oriented questions that invite a partner to critique one's own thinking.

Other-monitoring

Other-oriented statements, questions and responses that represent an attempt to engage with and understand a partner's thinking.

Results

Metacognitive Strategies

Belinda's skill in planning, monitoring and evaluating her progress, and dealing with obstacles is illustrated in the following macroscopic analysis of the problem solving protocol.

Pre-Transcript: Planning. At the start of the lesson Belinda consulted the first author about using the compound interest formula $A = P(1+i)^n$, already familiar to her in calculations involving interest compounding annually. She sought confirmation that for interest compounding monthly she should divide the annual interest rate by twelve, and multiply the number of compounding steps by twelve. The observer also pointed out an error: Belinda had written the formula incorrectly as $A = P + (1+i)^n$. Before beginning work on the problem, Belinda had therefore prepared an alternative strategy to calculate the returns for both institutions—particularly Institution 2, with the unfamiliar situation of interest compounding monthly—and planned to use her calculator as a device for checking answers produced by uncertain spreadsheet methods. The error in her formula and her evident desire to match parts of the formula to the spreadsheet columns also suggest that she was searching for connections between the two alternative representations of the problem.

Segment 1: Error Recognition. Belinda and Louise both chose an investment term of twenty years as being suitably 'long term' and worked separately during the first few minutes. Before creating any spreadsheets Belinda used her calculator and the compound interest formula to find the expected returns from Institution 1 ($10545.09) and Institution 2 ($10892.55). These were the results against which she was to judge the accuracy and reasonableness of her spreadsheet methods. However, although she had correctly calculated 240 compounding steps and a monthly interest rate of 0.01 (1%) for Institution 2, Belinda had then matched her spreadsheet interest formula (Column C) to the $(1+i)$ element in the compound interest formula by adding one to the monthly interest rate, giving an effective rate of 1.01 (101%) for each compounding period (Figure 2, cell C2). This gave a return of $\$5.84 \times 10^{75}$, a result judged by Belinda to be so unreasonable, and so different from the answer previously returned by her calculator, that she immediately suspected she had made an error:

B: Do you get a horrifically huge number on the ...? (chanting) I hate this, I hate this, I ... hate, I hate this.

	A	B	C	D
1	Month	Principal	Interest	Amount
2	1	1000	=B2*1.01	=B2+C2
3	=A2+1	=D2	fill downm	fill downm
4	fill down to 240 mthsm	fill downm		

Figure 2 Belinda's spreadsheet for Institution 2

For the remainder of the lesson, Belinda directed her energies towards resolving this impasse. She clarified her thinking by explaining her strategy to others, carefully evaluated the appropriateness of strategies suggested by the teacher and her peers, and checked her own and other people's results for accuracy and sense. Most importantly, her insistence on understanding the problem rather than simply producing an answer prevented her from being misled by the incorrect explanations offered by those whose help she sought. A resolution was eventually achieved when the teacher intervened (Figure 3).

Segment 2: Clarify Goal

B asked L how to set up two spreadsheets side by side, so that returns from the two institutions could be compared. L argued this was unnecessary—just compare final returns after 20 years. B pointed out the goal was to find when the return from one institution overtook the other.

Segment 3: Strategy Justification

B moved to help third (male) student, R, who had made the error of using Institution 2's annual interest rate (12% or 0.12) instead of monthly rate (1% or 0.01). B's pointed out his error and explained her own strategy:

B: Because it's compounding interest—(confidently) this is compounding annually so it doesn't matter. This one is compounding monthly so you have to take into account the number of times you compound it in a year.

And later:

R: What interest rate did you use for the second one?

B: Point zero one. It's not actually point zero one, it's [inaudible]. See, compound interest, you've got to add one to it.

Segment 4: Seek Help from Teacher

B still troubled by the incongruity of her answer—decided to check her reasoning with the teacher (T). He confirmed that B's approach of multiplying the principal at the beginning of each compounding period by $(1+i)$ was potentially legitimate (Column C on B's spreadsheet), and pointed out that a fourth column would then be unnecessary as this single operation combined the two separate calculations of Columns C and D. Unsure whether the $(1+i)$ approach would work for interest compounding monthly, T then explained the conventional four column method (Figure 1). Unfortunately, the explanation contained some inconsistencies misinterpreted by B as endorsing the strategy initially used by R: that is, not dividing annual interest rate by twelve, even though the rate is compounding monthly. This conflicted with B's understanding of monthly compounding; moreover, the return on investment did not tally with that given by her alternative, calculator, method. Although B accepted T's advice that it was not necessary to 'add one', the 'divide by twelve' strategy had become a source of doubt, and B was not willing to accept an explanation she did not understand:

B: (to T) Right. Yeah, OK. (T leaves) ... (to herself) I still don't understand.

Segment 5: Seek Help from Peer

B now turned for help to her friend, L. Unfortunately, L had used the same incorrect method for calculating the return for Institution 2 as R. Despite her good intentions, L was not able to elaborate her method clearly enough for B to understand:

L: [...] In twenty years that's the final investment (pointing at screen); in 22 months that's the final investment, and it's more. OK, 22nd month, in 22 months it'll start to be a better investment. Does that make sense to you?

B: (sincerely) Yeah it does. I just don't understand how you do it.

However, B was able to obtain the information she sought by asking L to display spreadsheet cell C2 containing the interest formula '=B2*0.12'. To B's surprise, this was the formula whose flaws she had so confidently pointed out to R (Segment 3), and the same formula that seemed, to B, to be endorsed by T (Segment 4). B checked her understanding by interpreting the formula in her own words—a move that triggered in L the realisation of her error:

B: Oh my God! So—what do you mean to tell me that all you do is if you're given a percent, if you're given an interest rate, that all you do is you put it in, but then you've got to make sure that instead of doing down [i.e. filling down] the years, you've got to do it of the years, multiply by [twelve]—

L: (slowly, emphasising each word) I have done it wrong!

Segment 6: Check Alternative Strategies

B temporarily set aside her doubts regarding L's spreadsheet formula for Institution 2 in order to test her conviction that the return on investment could be found by using either calculator/compound interest formula method or spreadsheet/iterative method. Using the relevant data from L's spreadsheet, B found both methods produced the same result: 6.49×10^{14}. Although now satisfied that the alternative problem representations were reconcilable, B was still concerned that the answer did not make sense.

Segment 7: Resolution

As B began to explain her doubts to L, the teacher again intervened and quickly spotted the error that had been troubling B since their earlier conversation:

T: [...] that should be point 01, not point 12.

L: I knew I did it wrong! [...] Oh yeah, it takes, it takes—right, I understand.

B: (singing) I understand! (laughs)

Figure 3 Macroscopic analysis of interest rate protocol

Metacognitive Function of Dialogue

Belinda's attempts to articulate her methods and monitor her progress took place in the context of her interactions with other students and the teacher, all of whom contributed to joint metacognitive activity. Some insight into the nature of the social interactions that supported this activity may be gained by examining conversational moves within the protocol that had both a metacognitive function (New Idea or Assessment) and a transactive structure (self- or other-oriented statement, question or response). Of the 146 conversational moves made by all speakers, twenty were of this type. Table 1 shows that speakers monitored their own and each other's thinking by asking a partner to comment on their work, offering critiques of their own or a partner's strategies, and elaborating on their ideas for the benefit of a partner.

Table 1 Metacognitive Purposes Served by Transactive Dialogue

Metacognitive Function	Transactive Structure	Frequency
Assessment: accuracy or reasonableness of result	Self-oriented question (request critique)	3
Assessment: appropriateness of strategy	Other-oriented statement (critique)	10
	Self-oriented statement (critique)	2
	Self-oriented question (request critique)	1
New Idea: propose strategy	Self-oriented statement (clarification, elaboration, justification)	4

Collaborative Interactions

Because participants had unequal opportunities to contribute to the discussion, the six types of transacts were counted as proportions (frequency divided by the total number of conversational turns taken by that person) as well as frequencies. Table 2 gives details of the transacts produced by Belinda, Louise, Rob and the teacher. Although each student's dialogue contained a similar proportions of total transacts, only Belinda demonstrated the balance of self-disclosure, requesting feedback, and monitoring others' thinking that indicates a collaborative discussion style.

Table 2 Collaborative Quality of Transactive Dialogue

	Frequencies (Proportions)			
Transact Grouping	Belinda	Louise	Rob	Teacher
• Self-disclosure	7 (.106)	7 (.175)	0 (.000)	4 (.250)
• Feedback request	3 (.045)	2 (.050)	1 (.050)	0 (.000)
• Other-monitoring	12 (.182)	2 (.050)	5 (.250)	5 (.313)
Total Transacts	22 (.333)	11 (.275)	6 (.300)	9 (.563)

Summary and Conclusions

This case study of informal peer interaction has found some evidence of metacognitive behaviour, such as checking and justification, previously observed to be lacking in small group work in mathematics (Stacey, 1992). One student, Belinda, showed considerable purpose and skill in planning her solution method (identifying the goal, formulating alternative representations of the problem, selecting a strategy), monitoring and evaluating her progress (assessing results and strategies) and dealing with obstacles (recognising an impasse, trying an alternative strategy, seeking help). However, other students did not display the same level of awareness and self-monitoring: for example, Louise was oblivious to a major strategic error because she failed to appreciate the significance of the unrealistic return on investment she had calculated for Institution 2.

Although Belinda's solution attempt was largely self-regulated, she also sought social support by inviting feedback from others when her progress faltered. These interactions were marked by her willingness to explain and justify her own ideas, and to explore her partner's reasoning—signs of the mutual engagement thought to characterise collaboration.

Because of the limited scope of this study some caution should be exercised in interpreting the findings. Nevertheless, the results raise some issues that have implications for mathematics teachers. First, it appears that Belinda's actions were driven by her desire to *understand*: she was not satisfied until she had reconciled her two solution methods, found the source of her own and her partners' errors, and arrived at an answer that made sense. Learning to think mathematically involves understanding as well as mastering facts and procedures, and students may need to acquire a disposition for sense-making if they are to successfully apply metacognitive strategies. Second, metacognitive activity and collaborative interaction do not in themselves guarantee a mathematically productive outcome. A student may be very articulate and confident in justifying an incorrect strategy (as was Belinda in Segment 3) or wrongly rejecting a partner's strategy, and peers working collaboratively may yet be unable to overcome an obstacle. The challenge for the teacher is to find ways of intervening that help students work together to extricate themselves from their difficulties.

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